Inferring Causation from Correlation in Sparse Networks

Alexander Niederbühl[1] A.NIEDERBUEHL@GMAIL.COM
Volker Pernice[1*] VOLKER.PERNICE@LPS.ENS.FR
Stefan Rotter[1] STEFAN.ROTTER@BIOLOGIE.UNI-FREIBURG.DE
[1] *Bernstein Center Freiburg and Faculty of Biology, University of Freiburg*

Abstract

We discuss the applicability of a recently proposed method to reconstruct cerebral network structure (Pernice and Rotter, 2013) on the data supplied in the Neural Connectomics Challenge. This method is based on a model of linear dynamics which provides a relation between activity covariances and the underlying network. Even for very limited sampling resolution, an estimate of the directed connections can be obtained under the assumption of sparse connectivity, if indirect connections contribute significantly to the covariance matrix. Surprisingly, and possibly due to the low importance of indirect connections in the provided data, the sparsest estimator resulted in only limited performance. Our best estimation relies on the inverse covariance matrix which has been adapted based on insights gained from the model.

1. Introduction

The method recently proposed in Pernice and Rotter (2013) presents a novel approach to the classical problem of inferring causality from correlations, which does not depend on temporal precedence like Granger causality (Bressler and Seth, 2011) or Transfer Entropy (Schreiber, 2000). Under the assumption of a sparsely connected underlying network, information from indirect connections can be exploited to convert the inverse covariance matrix to a directed estimator. This can be done for covariance functions in the frequency domain, but also for zero-lag covariances arising from processes that are faster than the experimental time resolution.

2. Method

We approximate the cerebral network as a linear system with recurrent feedback. Considering a network of n nodes let $x(t)$ be the n-dimensional vector of continuous-time signals measured from these nodes and $X(\omega)$ its Fourier transform. The interactions between nodes at frequency ω are described by the matrix $G(\omega)$ such that the dynamics of the network is governed by the equation

$$X(\omega) = G(\omega)X(\omega) + V(\omega). \tag{1}$$

The term $V(\omega)$ can be interpreted as the activity of the nodes independent of the recurrent feedback due to the network. By solving Eq. (1) for $X(\omega)$ we get $X(\omega) = \big(\mathbb{1} - G(\omega)\big)^{-1}V(\omega)$. The cross-spectral matrix is then given as

$$\hat{C}(\omega) = \mathbb{E}[X(\omega)X^*(\omega)] = \big(\mathbb{1} - G(\omega)\big)^{-1}Y(\omega)\big(\mathbb{1} - G^*(\omega)\big)^{-1}, \tag{2}$$

[*] Current Adress: Laboratoire de Physique Statistique, ENS Paris

with the real diagonal matrix $Y(\omega) = \mathbb{E}\big[V(\omega)V^*(\omega)\big]$. Because the rows of $|\mathbb{1} - G(\omega)|^{-1}$ and the diagonal elements of $\sqrt{Y}$ are multiplicatively related to each other we can set $Y = \mathbb{1}$ without loosing descriptive power.

Since $\hat{C}(\omega)$ is the Fourier transform of $C(\tau) = \mathbb{E}\left[x(t)x(t+\tau)^{\mathrm{T}}\right]$ with time lag τ, we can use only $\hat{C}(0) = C(0) \equiv C$ if the sampling rate so slow that we only observe covariances at time lag zero. In the following, we only discuss this reduced form of the model, which is equivalent to a structural equation model (McIntosh and Gonzalez-Lima, 1994). However, if there is information in the covariance function at other lags, the same method can be applied seperately on any $\hat{C}(\omega)$.

In this framework C is explained in terms of the adjacency matrix G and can be written as power series decomposition $C = \sum_{n,m=0}^{\infty} G^n (G^T)^m$, which illustrates that an entry of C includes the influence between nodes over all possible paths (Pernice et al., 2011). As is well known (Dahlhaus et al., 1997), effects of indirect connections are reduced in the inverse covariance matrix, here

$$C^{-1} = \mathbb{1} - G - G^T + G^T G \tag{3}$$

where the remaining spurious interactions are given by the term $G^T G$. Eq. (3) cannot be uniquely solved for G, as can be easily seen by writing $C^{-1} = B^T B$ with $B = (\mathbb{1} - G)$. We can substitute B with any matrix UB, as long as $U^T U = \mathbb{1}$, without changing the covariance structure. If it is reasonable to assume a sparse underlying network, as in the case of neural connections on a cellular level, this information can be used to resolve the ambiguity by stochastic minimization of the L_1 norm of the columns of B over all unitary transformations U.

This method could recover the correct network structure when applied to covariance matrices given by Eq. (2), simulated dynamics of leaky-integrate-and-fire neurons (Pernice and Rotter, 2013) and coupled autoregressive processes (not shown).

3. Neural Connectomics Challenge

On the data provided in context of the Neural Connectomics Challenge, most signal information was extracted with fast nonnegative deconvolution (Vogelstein et al., 2010). Because of the slow sampling rate, we assumed that much of the information which can be extracted from the cross-covariance function is contained in the value at time shift zero. However, the minimization of the L_1 norm of B did not improve the result. Reasons could be either strong nonlinear effects that are not captured by the dynamic model, the low signal-to-noise-ratio, or the influence of indirect connections being too weak. Instead, we based the reconstruction on C and C^{-1}. Since only excitatory connections are present, the term $G^T G$ is positive, and the estimator from Eq. (3) can be improved by using $|[C^{-1}]_-| \approx G + G^T$, by about 0.04 AUC. With the notion that causes precede their effects, we use the covariance C(T) at the time shift of one sampling period T, to get a measure of the direction of the interactions. We consider a nonzero covariance between two nodes a necessary condition for a connection, which is why our final estimator for G is a multiplicative combination of C, $|[C^{-1}]_-|$ and $C(T)$, which performed less than 0.005 AUC worse than the winning solution when applied on the test data set of the challenge (Table 1).

Table 1: Result of the winning team in comparison to the approach described in Section 3.

Rank	Team Name	AUC
1	AAAGV	0.94161
7	Alexander N & vopern	0.93666

Acknowledgements

This work was supported by the BrainLinks-BrainTools Cluster of Excellence funded by the German Research Foundation (DFG, grant no. EXC 1086).

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